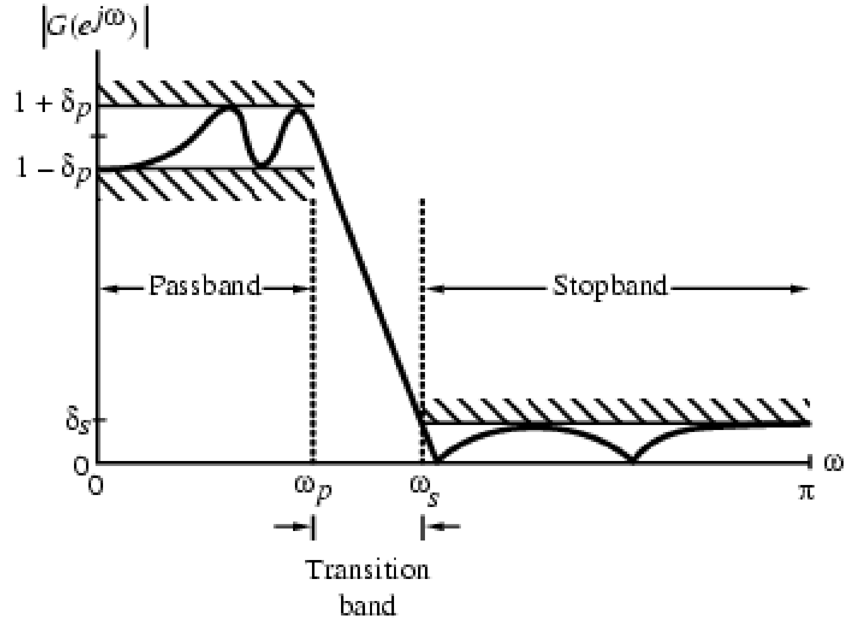
Digital Filter Specifications

The magnitude response of a digital lowpass filter may be given as indicated below



Digital Filter Specifications

Filter specification parameters

- ω_p - **passband edge frequency**
- ω_s - **stopband edge frequency**
- δ_p - **peak ripple value** in the passband
- δ_s - **peak ripple value** in the stopband

Digital Filter Specifications

- Practical specifications are often given in terms of **loss function (in dB)**
- $$\mathrm{G}(\omega) = -20\log_{10}\left|G(e^{j\omega})\right|$$
- **Peak passband ripple**

$$\alpha_p = -20\log_{10}(1-\delta_p) \quad \text{dB}$$

- **Minimum stopband attenuation**

$$\alpha_s = -20\log_{10}(\delta_s) \, \text{dB}$$

Digital Filter Specifications

- In practice, passband edge frequency F_p and stopband edge frequency F_s are specified in Hz
- For digital filter design, normalized bandedge frequencies need to be computed from specifications in Hz using

$$\omega_p = \frac{\Omega_p}{F_T} = \frac{2\pi F_p}{F_T} = 2\pi F_p T$$

$$\omega_s = \frac{\Omega_s}{F_T} = \frac{2\pi F_s}{F_T} = 2\pi F_s T$$

Digital Filter Specifications

- Example - Let $F_p = 7$ kHz, $F_s = 3$ kHz, and $F_T = 25$ kHz
- Then

$$\omega_p = \frac{2\pi(7\times10^3)}{25\times10^3} = 0.56\pi$$

$$\omega_s = \frac{2\pi(3\times10^3)}{25\times10^3} = 0.24\pi$$

Selection of Filter Type

- The transfer function $H(z)$ meeting the specifications must be a causal transfer function
- For IIR real digital filter the transfer function is a real rational function of

$$H(z)=\frac{p_0+p_1 z^{-1}+p_2 z^{-2}+\cdots+p_M z^{-M}}{d_0+d_1 z^{-1}+d_2 z^{-2}+\cdots+d_N z^{-N}}$$

- $H(z)$ must be stable and of lowest order N or M for reduced computational complexity

Selection of Filter Type

- FIR real digital filter transfer function is a polynomial in z^{-1} (order *N*) with real coefficients

$$H(z) = \sum_{n=0}^{N} h[n] z^{-n}$$

- For reduced computational complexity, degree *N* of *H*(*z*) must be as small as possible
- If a linear phase is desired then we must have:

$$h[n] = \pm h[N-n]$$

- (More on this later)

Constraints and Performance Measures

- BIBO stability:
  - If $|x(n)| < \infty$, it is required that $|y(n)| < \infty$.
  - Poles should be inside unit circle: $|p_j| < 1$ (for causal systems where h(n)=0 for n < 0.)
- Dynamic range overflow:
  - Intermediate or final result should not cause overflow
- Quantization error:
  - Should be bounded.
  - Should not cause instability.
- Speed:
  - Throughput rate and number of operations per data sample
- Hardware:
  - Memory I/O, address calculation, register footprint, special hardware, etc.

DIFFERENCE BETWEEN IIR FILTER AND FIR FILTER

FIR Digital Filter

- Let {h[n}: impulse response
  {x(n)}: input,
  {y(n)}: output
- Finite impulse response (FIR) filter:

$$y(n) = \sum_{j=0}^{J-1} h(j)x(n-j)$$

IIR Digital Filter

- Infinite impulse response (IIR) filter

$$y(n) = \sum_{i=1}^{P} a(i)y(n-i) + \sum_{k=0}^{Q} b(k)x(n-k)$$

$$\frac{Y(z)}{X(z)} = \frac{\sum_{m=0}^{Q} b(m)z^{-m}}{1+\sum_{k=1}^{P} a(k)z^{-k}} = \frac{B(z)V(z)}{A(z)V(z)}$$

FIR Digital Filter

- Impulse input:

  if $x(n)=\delta(n)$, $y(n)=h(n)$ is the impulse response that has finite extent.

- Computation is the same as convolution.

IIR Digital Filter

- The length of $\{y(n)\}$ may be infinite!
- Stability concerns:
  - The magnitude of $y(n)$ may become infinity even if all $x(n)$ are finite!
  - coefficient values,
  - quantization error

FIR Digital Filter

- FIR filter can be implemented using direct form or fast convolution methods like FFT ,hence STABLE.
- Realized by Non-Recursive methods.

IIR Digital Filter

- IIR filters are often factored into products (cascade realization) or sum (parallel realization) of 1^{st} order or 2^{nd} order sections due to numerical concerns(Manual Calculation only possible)
- Realised by Recursive(Feedback) methods.

FIR Digital Filter

- They have LINEAR PHASE.
- Less susceptible to Noise.
- To design we have

a)Park Mc Clellan's method.

b)Fourier Series method.

c)Frequency Sampling OR Inverse Fourier Transform method.

d)Window technique.

E.g. Rectangular,Hamming,Hanning,Bartlett,Blackmann,Kaiser Windows.

e)Minimax or Optimal Filter Design.

IIR Digital Filter

- They don't have linear phase & hence are used at places where phase distortion is tolerable.
- More susceptible to Noise.
- To design we have

a)Impulse Invarience method.

b)Bilinear Transformation method.

c)Backward difference method.

FIR Digital Filter

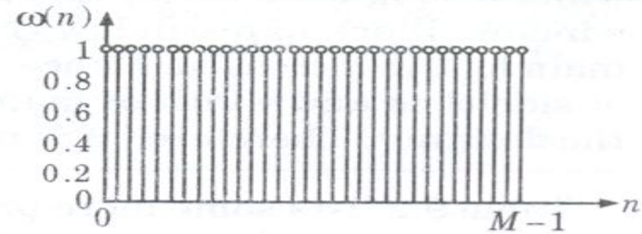
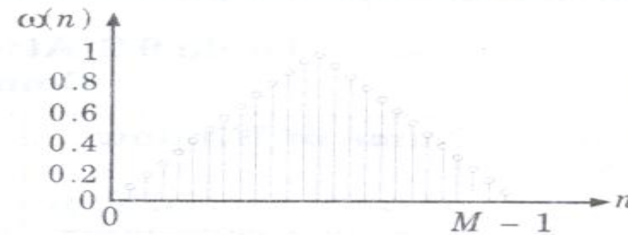
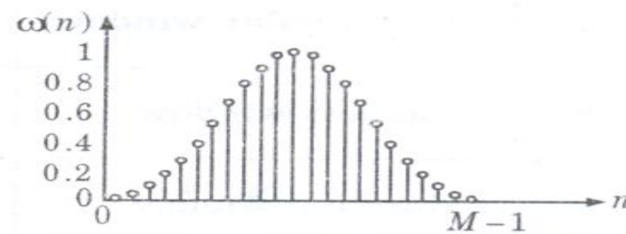
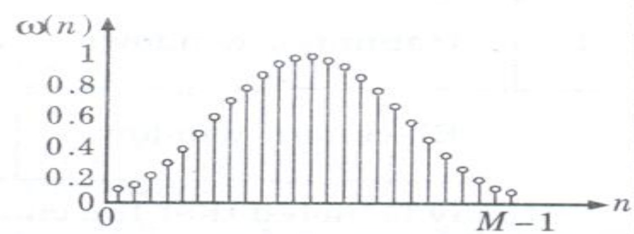
- Storage Requirements & Arithmetic operation is more here.
- Greater Flexibility to control the shape of their Magnitude response & Realization Efficiency.

IIR Digital Filter

- Storage Requirements & Arithmetic operation is less.
- Less Flexibility to control the shape of their Magnitude response.
- Often derived from analog filters

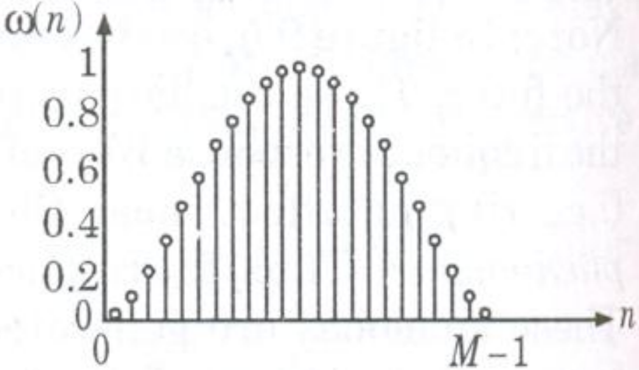
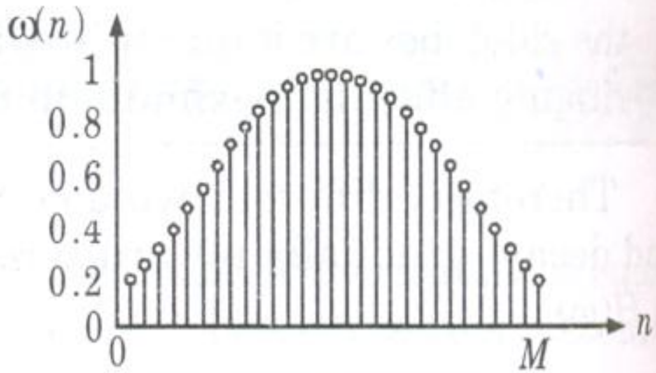
Various other window functions

Table 9.1 Various window functions and their corresponding shapes

Sr. No.	Name of Window	Time-domain sequence, $\omega(n)$, $0 \le n \le M-1$	Shape of window function
1.	Rectangular	1	
2.	Bartlett (triangular)	$1-\dfrac{2\left\lvert n-\dfrac{M-1}{2}\right\rvert}{M-1}$	
3.	Blackman	$0.42–05\cos\dfrac{2\pi n}{M-1}+0.08\cos\dfrac{4\pi n}{M-1}$	
4.	Hanning	$0.54-0.46\cos\dfrac{2\pi n}{M-1}$	

(Contd..)

Various other window functions

5.	Hanning	$\frac{1}{2}\left(1-\cos\frac{2\pi n}{M-1}\right)$	
6.	Kaiser	$\frac{I_0\left\{\beta\left[1-\left(\frac{n-\alpha}{\alpha}\right)^2\right]^{\frac{1}{2}}\right\}}{I_0(\beta)}$	

Comparative Study for Trade Off between Attenuation of Sidelobes & Transition Width of main Lobe.

Table [illegible].2 Attenuation and Transition Widths of Commonly Used Windows

S. No.	Name of Window	Transition width of the main lobe	Minimum stopband attenuation	Relative amplitude of sidelobe
1.	Rectangular window	$\frac{4\pi}{M+1}$	– 21 dB	– 13 dB
2.	Bartlett window	$\frac{8\pi}{M}$	– 25 dB	– 25 dB
3.	Hanning window	$\frac{8\pi}{M}$	– 44 dB	– 31 dB
4.	Hamming window	$\frac{8\pi}{M}$	– 53 dB	– 41 dB
5.	Blackman window	$\frac{12\pi}{M}$	– 74 dB	– 57 dB

It may be noted that the characteristics of Kaiser window have not been mentioned

FIR Filter Design: Rectangular Window

- Let w(n)=Rectangular Window Function,
- Where
- w(n)=1 $0 \le n \le M-1$

hd(n)=Infinite Input Sequence(Arbitrary),&

h(n)=Finite Truncated Impulse Response.

Then

$h(n)=hd(n) \times w(n)$

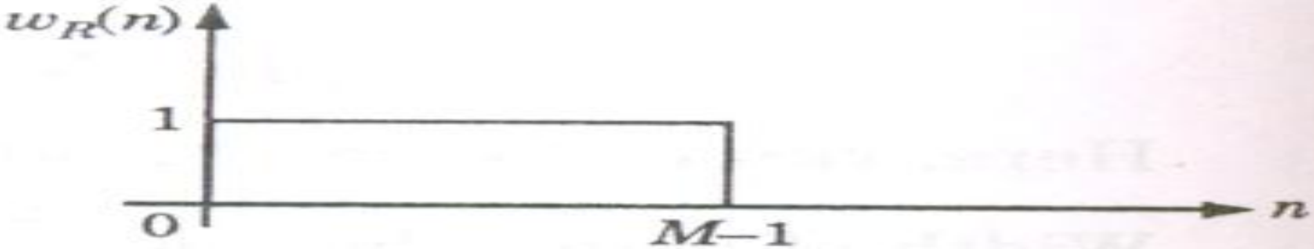


Fig. 9.4 *Rectangular window.*

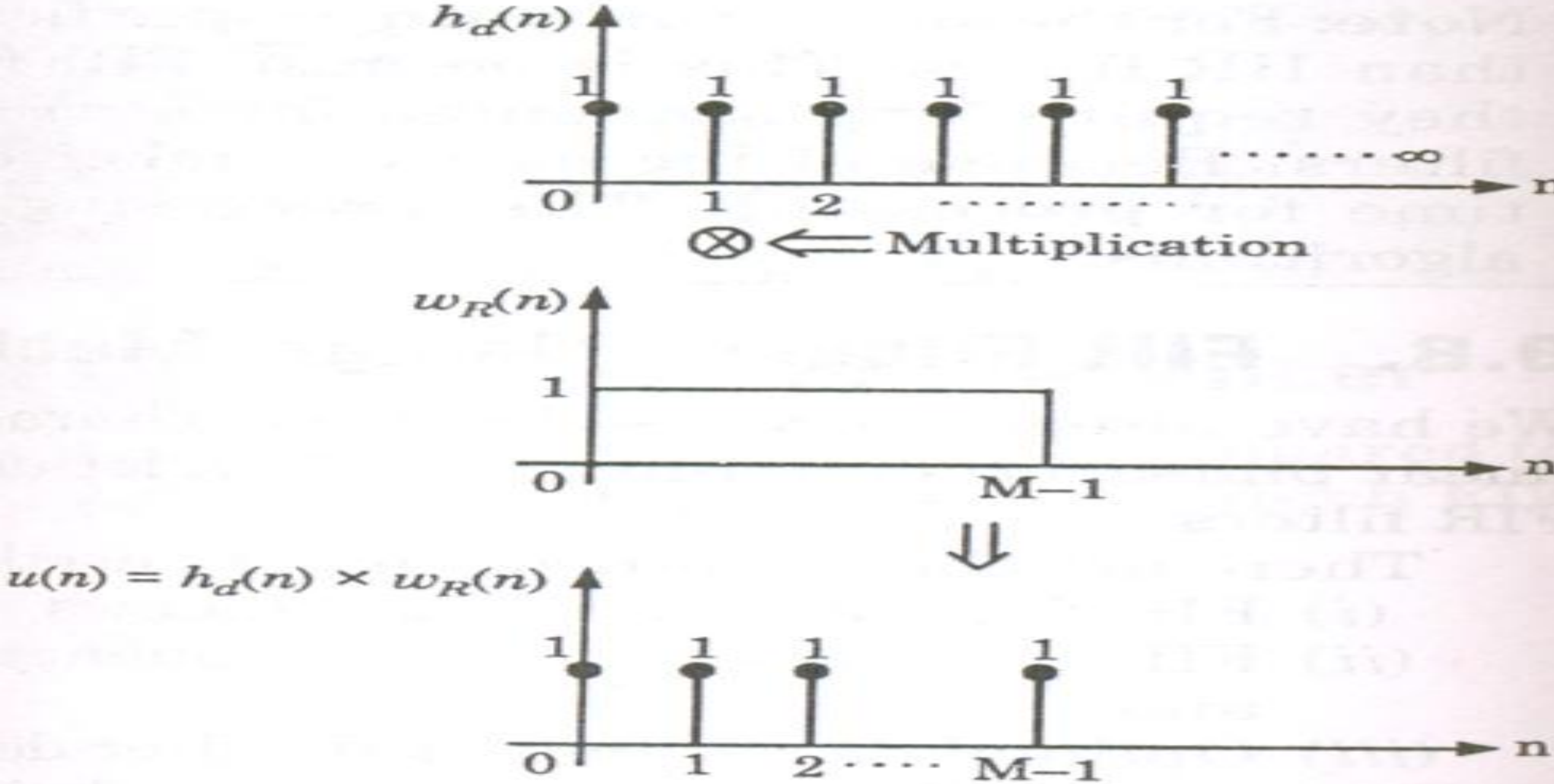


Fig. 9.5. *Truncation process*

for $n = 0, 1, \ldots, M-1$

Gibbs Phenomenon:Ringing Effect/Oscillatory Behaviour due to Sidelobes(generated owing to the sharp cut-off/abrupt discontinuity) in the Frequency Response of the window Function

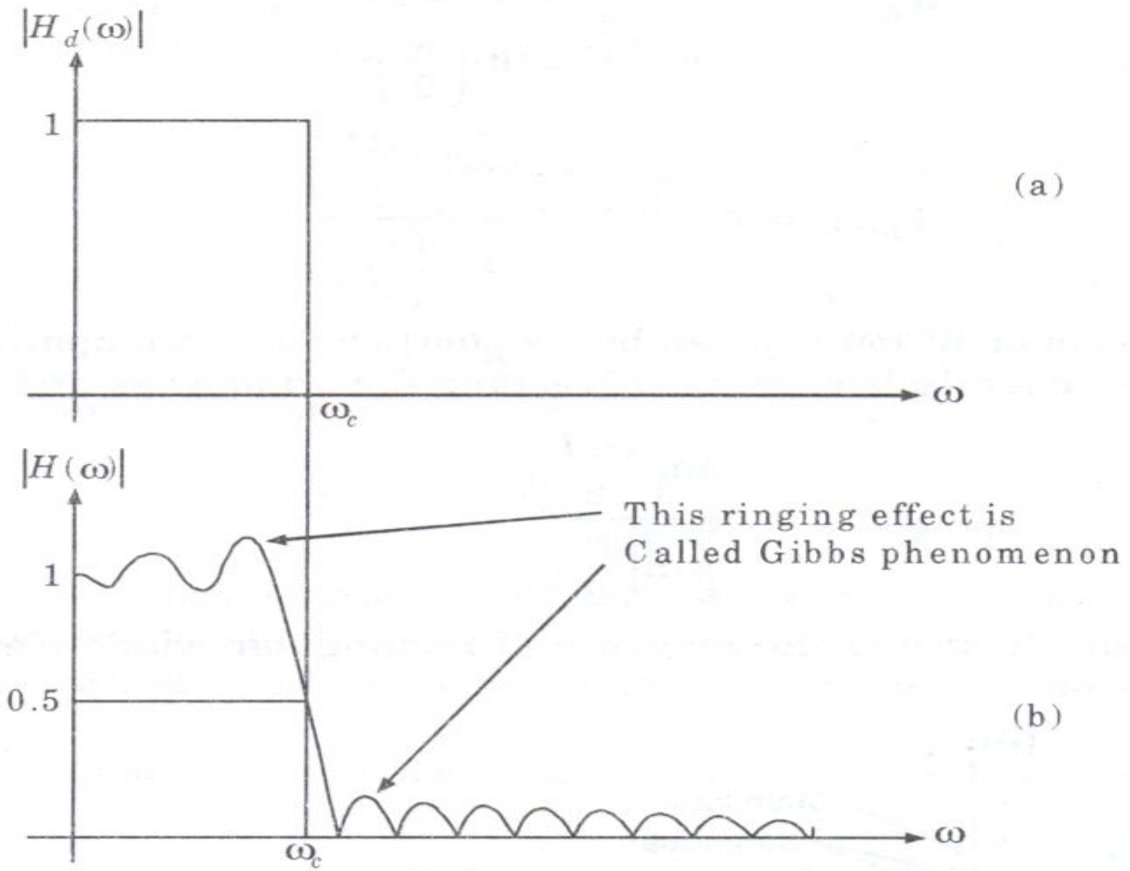


Fig. 9.6. *(a) The desired frequency response $H_d(\omega)$ (b) The frequency response of FIR filter obtained by windowing. It has smoothing and ringing effect because of windowing.*